Intervallo di Confidenza

Per intervallo di confidenza si intende trovare un intervallo all'interno del quale, con una certa approssimazione (1-α) si trova il parametro incognito. L'intervallo può essere a una coda (dx o sx) o a due code.

I.C. su media μ

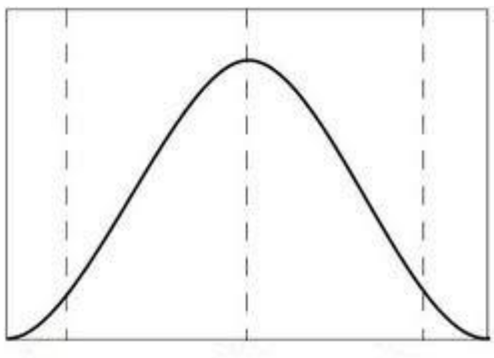
Con σ^2nota:

2code: $\bar{X} - \frac{\sigma}{\sqrt{n}} Z_{1-\frac{\alpha}{2}} < \mu < \bar{X} + \frac{\sigma}{\sqrt{n}} Z_{1-\frac{\alpha}{2}}$ I.C.: $[\bar{X} - \frac{\sigma}{\sqrt{n}} Z_{1-\frac{\alpha}{2}}\ ;\ \bar{X} + \frac{\sigma}{\sqrt{n}} Z_{1-\frac{\alpha}{2}}]$

1 coda: sx: $\mu > \bar{X} - \frac{\sigma}{\sqrt{n}} Z_{1-\alpha}$ I.C.: $[\bar{X} - \frac{\sigma}{\sqrt{n}} Z_{1-\alpha}\ ;\ +\infty)$

2 code: dx: $\mu < \bar{X} + \frac{\sigma}{\sqrt{n}} Z_{1-\alpha}$ I.C.: $(-\infty\ ;\ \bar{X} + \frac{\sigma}{\sqrt{n}} Z_{1-\alpha}]$

$-Z_{1-\frac{\alpha}{2}}$ $Z_{1-\frac{\alpha}{2}}$

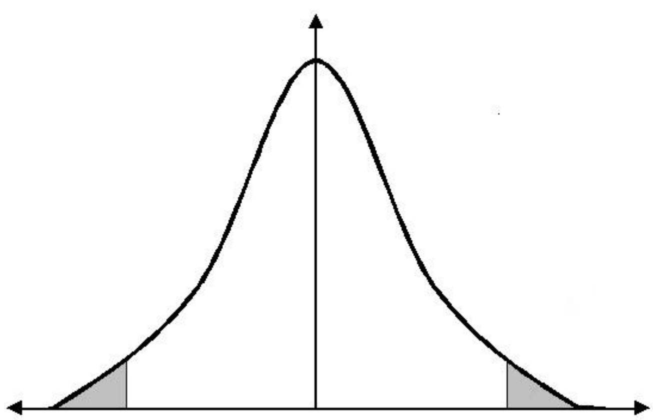


Con σ^2 non nota:

2code: $\bar{X} - \frac{S_n}{\sqrt{n}} t_{1-\frac{\alpha}{2}} < \mu < \bar{X} + \frac{S_n}{\sqrt{n}} t_{1-\frac{\alpha}{2}}$ I.C.: $[\bar{X} - \frac{S_n}{\sqrt{n}} t_{1-\frac{\alpha}{2}}\ ;\ \bar{X} + \frac{S_n}{\sqrt{n}} t_{1-\frac{\alpha}{2}}]$

1 coda: sx: $\mu > \bar{X} - \frac{S_n}{\sqrt{n}} t_{1-\alpha}$ I.C.: $[\bar{X} - \frac{S_n}{\sqrt{n}} t\ ;\ +\infty)$

2 code: dx: $\mu < \bar{X} + \frac{S_n}{\sqrt{n}} t_{1-\alpha}$ I.C.: $(-\infty\ ;\ \bar{X} + \frac{S_n}{\sqrt{n}} t_{1-\alpha}]$



$-t_{1-\frac{\alpha}{2}}$ $t_{1-\frac{\alpha}{2}}$

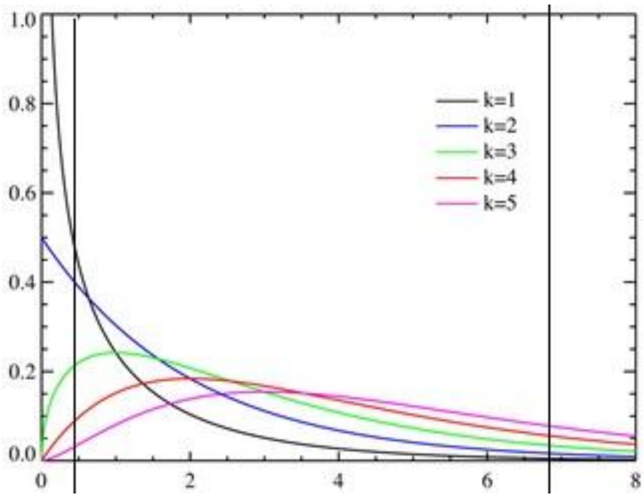
I.C. su varianza σ^2

Con μ nota:

$$\frac{nS_n}{\chi^2_{1-\frac{\alpha}{2},\ n}} < \sigma^2 < \frac{nS_n}{\chi^2_{\frac{\alpha}{2},\ n}} \qquad \text{I.C.:}\left[\frac{nS_n}{\chi^2_{1-\frac{\alpha}{2},\ n}}\ ;\ \frac{nS_n}{\chi^2_{\frac{\alpha}{2},\ n}}\right]$$

Con μ non nota:

$$\frac{(n-1)S_n}{\chi^2_{1-\frac{\alpha}{2},\ n-1}} < \sigma^2 < \frac{(n-1)S_n}{\chi^2_{\frac{\alpha}{2},\ n-1}} \qquad \text{I.C.:}\left[\frac{(n-1)S_n}{\chi^2_{1-\frac{\alpha}{2},\ n-1}}\ ;\ \frac{(n-1)S_n}{\chi^2_{\frac{\alpha}{2},\ n-1}}\right]$$



$\chi^2_{1-\frac{\alpha}{2},\ n-1}$ $\chi^2_{\frac{\alpha}{2},\ n-1}$

Dati $X_1, \dots, X_n \sim \mathrm{N}(\mu_x, \sigma_x^2)$ con $\mu_x, \sigma_x^2, \mu_y, \sigma_y^2$ non note

$Y_1, \dots, Y_m \sim \mathrm{N}(\mu_y, \sigma_y^2)$

I.C. su $\mu_x - \mu_y$

$$\frac{\bar{X}_n - \bar{Y}_m - (\mu_x - \mu_y)}{\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}} \sim \mathrm{N}(0,1)$$

$$\bar{X}_n - \bar{Y}_m - Z_{\frac{\alpha}{2}}\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}} < \mu_x - \mu_y < \bar{X}_n - \bar{Y}_m + Z_{\frac{\alpha}{2}}\sqrt{\frac{S_x^2}{n} + \frac{S_y^2}{m}}$$

In generale, però, non possiamo dire che $(\mu_x - \mu_y) \sim \mathrm{N}(0{,}1)$

Se $\boldsymbol{\sigma_x^2 = \sigma_y^2 = \sigma^2}$ incognito:

A. $\frac{\bar{X}_n - \bar{Y}_m - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma^2}{n} + \frac{\sigma^2}{m}}} \sim \mathrm{N}(0{,}1)$

B. $\frac{(n-1)S_x^2}{\sigma^2} \sim \chi^2_{n-1} + \frac{(m-1)S_y^2}{\sigma^2} \sim \chi^2_{m-1} \quad => \quad \sim \chi^2_{n+m-2}$

Se facciamo $\frac{A}{B} \sim \mathrm{T}(\mathrm{n+m-2})$ e indichiamo con $S_p^2 = \frac{(n-1)S_x^2 + (m-1)S_y^2}{n+m-2}$ varianza campionaria Pooled

I.C. simmetrico per $\mu_x - \mu_y$:

$$\bar{X}_n - \bar{Y}_m - t_{\frac{\alpha}{2},n+m-2}\sqrt{\frac{1}{n} + \frac{1}{m}} * S_p < \mu_x - \mu_y < \bar{X}_n - \bar{Y}_m + t_{\frac{\alpha}{2},n+m-2}\sqrt{\frac{1}{n} + \frac{1}{m}} * S_p$$

Dato un campione bernoulliano numeroso:

$X_1, \dots, X_n \sim \mathrm{Be}(p)$

$$\bar{X}_n - z_{\frac{\alpha}{2}} \frac{\sqrt{\bar{X}(1-\bar{X})}}{\sqrt{n}} < p < \bar{X}_n + z_{\frac{\alpha}{2}} \frac{\sqrt{\bar{X}(1-\bar{X})}}{\sqrt{n}}$$

Indichiamo con $|I_n| = \mathrm{lunghezza\ di\ I_n} = 2z_{\frac{\alpha}{2}} \frac{\sqrt{\bar{X}(1-\bar{X})}}{\sqrt{n}}$

Ma dato che $0 < \bar{X}(1-\bar{X}) \leq \frac{1}{4}$

$|I_n| < \frac{z_{\frac{\alpha}{2}}}{\sqrt{n}}$